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Planck Units: Natural Units *and* the Key Equations in Physics

Charles William Johnson

Extract

The author examines the Planck Units whose formulae are based upon expressions of square roots. He considers the performance of those units as of the roots of the speed of light in a vacuum, fractal 1.731451582 and 5.475330657. He illustrates how different constant values appear for the Planck Units as of those root expressions. The Planck Units that have square root expressions in their equations by design cannot derive a single fractal numerical constant, when in fact square roots for any number produce two significant root numbers, just as cube roots produce three significant root numbers. The author explores the math behind the equations of the Planck Units in order to discern which fractal numerical constants are employed in each one. The author shows that those Planck Units based on square root expressions forward a single constant in their results, when mathematically there exist two variations due to the square root procedure. The deficiency in the Planck Units come from conceptualizing the metric system of a floating decimal as uniform for all equations, when the root variations produce seemingly contradictory fractal numerical results. The results or constants are not contradictory, but merely two options to the same mathematical procedure.

Part I

Planck Units

The Planck Units represent abstracted exercises in theoretical physics, as an attempt to create natural units of measurements. They reflect the minimal units of spacetime that may be measured. For example, the Planck Length represents the minimally abstracted length of spacetime that may be measured with a degree of accuracy. At least, this is how the Planck Units are generally presented by scientists in physics.

The main Planck Units reference length, mass, time, charge, and temperature. Beyond those basic five Planck Units there are derived units for

area, volume, momentum, energy, force, power, density, angular frequency, pressure, current, voltage and, impedance. [For a complete listing of the Planck Units and their values, consult the *Addendum* to this essay.]

The Planck Units that contain a square root expression within their formulae are: Planck length, mass, time, charge, temperature, momentum, energy, angular frequency, current and, voltage. The Planck Units that do not contain a square root expression within their terms, and which are not treated in this essay, are: Planck area, volume, force, power, density, pressure and impedance.

The Planck Units are generally presented with only one numerical value pertaining to each, supposedly representing a constant-like expression of the property of each unit. Consider the values given for the Planck Units that have a square root expression in their terms:

Planck length:	1.616252 <i>fractal scientific notation</i>
Planck mass:	2.17644
Planck time:	5.39124
Planck charge:	1.87554587
Planck temperature:	1.416785
Planck momentum:	6.52485
Planck energy:	1.9561
Planck angular frequency:	1.85487
Planck current:	3.4789
Planck voltage:	1.04295

It is said that Planck presented his values at the end of the nineteenth century to the scientific community without demonstrating how they were derived. From the analysis that follows, one may imagine why. The Planck Units that contain a square root expression within their terms, although assigned a single fractal numerical value each, are susceptible to producing two different values for the Planck Units themselves, *given the nature of the mathematical procedure of square roots*.

Square Roots

In earlier essays, I have been analyzing the nature of square root numbers, especially with regard to the speed of light in a vacuum [c]. When one employs the concept of the square root of a numerical value, two possibilities exist regarding their values. [www.earthmatrix.com/speed_of_light.html] For example, with regard to the square root of the speed of light in a vacuum, two distinct answers obtain as in the following expressions, as occurs with all square roots.

The square root of 2.99792458 in **scientific notation** equals **1.731451582**

The square root of 299792.458 in **kilometers** equals **547.5330657**

In other words, depending upon which fractal value is employed for the square root computation two distinct numerical values obtain. Although the first expression, 2.99792458 may reflect an actual measurement of the speed of light in a vacuum, it also reflects the customary scientific notation employed in physics. And, the second value, 299792.458 reflects the actual measured value of 299792,458 kilometers per second.

The value cited in the CODATA employs 299 792 458 **meters** per second.

The square root of 299 792 458 **meters** equals **17314.51582**

For the purposes of this study, it shall become significant to note that *both* the **scientific notation** expression and the value in **meters** for the speed of light in a vacuum produce a fractal value for the square root of 1.731451582. Whereas it is distinguishable that the value for the square root of the speed of light in a vacuum expressed in **millimeters** and **kilometers** yields a fractal numerical number of 5.475330657.

One might have surmised initially that both the values for the speed of light in meters and kilometers would be the same, given the basic design of a floating decimal placement within the metric system. However, with regard to the nature of floating decimals within the mathematical procedure of square roots, this is not the case. The placement of the decimal within the same fractal number as of the mathematical procedure of square roots produces two distinct roots for each number.

Similarly, the cube root of the speed of light produces three distinct fractal numerical values according to the decimal placement. The fourth root of the speed of light produces four distinct numerical values; and, the fifth root of the speed of light produces five distinct numerical values as roots. And infinitely so for whichever root value is chosen for the computations of a given number. Consider some of the roots for the fractal expressions of the speed of light:

The cube root of 2.99792458 equals **1.441916908**
 Cube root of **29.9792458** = **3.106515806**
 Cube root of **299.792458** = **6.692785417**
 Cube root of **2997.92458** = 14.41916908
 Cube root of **29979.2458** = 31.06515806
 Cube root of **299792.458** = 66.92785417

and so on, infinitely so in either direction of the decimal placement.

The fourth root of **0.0299792458** = 0.416107147
 Fourth root of **0.299792458** = 0.739954772
 Fourth root of **2.99792458** = **1.315846327**
 Fourth root of **29.9792458** = **2.339942447**
 Fourth root of **299.792458** = **4.161071475**
 Fourth root of **2997.92458** = **7.399547727**
 Fourth root of **29979.2458** = 13.15846327
 Fourth root of **299792.458** = 23.39942447
 Fourth root of **2997924.58** = 41.61071475
 Fourth root of **29979245.8** = 73.99547727
 Fourth root of **299792458.0** = 131.5846327

and so on, infinitely so in either direction of the decimal placement.

Note, the square root of the speed of light in **centimeters** would be **173145.1582**. Hence, we have:

The square root of 299792.458 **kilometers** equals **547.5330657**
 The square root of 299792458.0 **meters** equals **17314.51582**
 The square root of 29979245800.0 **centimeters** equals **173145.1582**
 The square root of 299792458000.0 **millimeters** equals **547533.0657**
 The square root of 29979245800000000.0 **nanometers** equals
547533065.7

As observed initially, the fractal values of the square root of the speed of light in a vacuum has two distinct fractal numerical values, 5.475330657 and 1.731451582. Therefore any equations in physics that contain the square root of light pose the possibility of two distinct answers as of those square root values, depending obviously, upon which metric is employed [nanometers, millimeters, centimeters, meters, kilometers, etc.]. This mathematical fact cannot be explained away with regard to the performance of the Planck Units that contain a mathematical procedure based on the square root of the speed of light in a vacuum.

So, the speed of light in a vacuum may be expressed in different divisional units of the metric system as nanometers, millimeters, centimeters, meters and, kilometers whereby all of these expressions reflect the exact same speed. Or, the values may expressed in scientific notation as a fractal 2.99792458 as significant numbers with the particular metric division following. The case remains that all of these measured values are reflective of the speed of light in a vacuum. Yet, due to the nature of square roots each particular metric division of measurement will have either a fractal 1.731451582 or a fractal 5.475330657 numerical expression, depending upon where the decimal place lies within said measurement.

When working then with square roots in the equations of physics, as in the Planck Units, the question arises as to which metric division shall be employed. Depending upon which one is employed in the equations, the results of the computational equations shall vary in their fractal numerical expressions. And, to propose the idea that a set of equations in physics such as the Planck Units produce a single fractal numerical result for each equation, yet are susceptible to having the varying measurements for the speed of light within their terms, is misleading, if not mistaken .

Once the formulae of the Planck Units are drawn up along the lines of square root expressions, there is no way to avoid this dilemma in the mathematical equations and their results. A choice was made in order to derive the Planck Unit of length of fractal 1.616252 [as of the root expression for c of 5.475330657]; a choice that could have equally been made to derive the unit length fractal 5.1110865 [as of the root for c of 1.731451582]. In this sense, 1.616252 is not the constant for Planck Unit length, but it is rather one of the relational constants; 5.1110865 is the other possible relational constant as of the square root of the speed of light within the formula for Planck length. Each one of the Planck Units based on square

roots has two *possible* fractal numerical answers [constants], depending upon the choice in the square root involved regarding the speed of light in a vacuum.

The aforesaid is not anything *sui generis* to the Planck Units, but rather represents a mathematical feature of square roots. All mathematical equations involving square roots may produce two specific results depending upon the decimal placement of the terms of those equations, i.e., depending upon the fractal numerical values of the square roots involved.

It is arbitrary and unadvisable to create equations that supposedly reflect physical and chemical constants of matter-energy based upon the square roots of the terms in this manner. In fact, it may be impossible to establish relationships of constancy that involve square roots [or any root term] due to the multiplicity of the products that become available from the mathematical procedure of roots. The ten cited Planck Units that contain square root procedures fall under this category of improbable singular constants.

Therefore, even though the 10-base metric system allows for the fractal expression of numerical values by floating the decimal place [as in 299 792 458 meters and 299792.458 kilometers], the roots of the metric expressions produce distinct value for those metric fractal values. This essay concerns itself with the square root and the corresponding values derived for the speed of light in a vacuum, as in root fractals **1.731451582** and **5.475330657**, in relation to the numerical values cited for certain Planck Units of measurement. The Planck Units of concern then are those that contain the square root expression within their equations.

In order to understand the alternate square root values for the square root of the speed of light in a vacuum, I translated the equations given for the Planck Units into their numerical computations. The previous numerical values for the ten Planck Units cited resulted from the computations employing the **root 5.475330657** for the speed of light ---irrespective of the specific fractal value of *the speed of light*.

The fractal numerical values for each of the ten Planck Units were produced by employing the fractal root 5.475330657. There is only one Planck Unit, that of Planck Charge, which employs the fractal root

1.731451582 value for its computations to derive the given constant in the literature.

One is immediately struck by the fact that the cited Planck Unit of length, for example, is expressed in **meters**, yet in order to derive that value, one must employ a square root value that corresponds to the **millimeter** or **kilometer** expression for the speed of light in order to be able to derive the fractal value 1.616252 meters. If one applies the square root fractal value of 1.731451582 that applies to the 299792458 **meter** expression for the speed of light, then an alternate value for the Planck Length is produced.

In this essay, for each formula given for a Planck Unit that contains an expression of the square root together with the speed of light within its confines, I substituted values based on fractal roots of 1.731451582 for the speed of light in substitution for the root 5.475330657 for c . Alternate numerical results obtain. The following alternate fractal numerical values were produced and are listed in the second column of the table below. If one were to employ the scientific notation value of 2.99792458 [scientific notation fractal] instead of the 299792458.0 meters/second value [CODATA] within the formulae given for each one of those Planck Units, then the fractal numerical values would be the same/similar fractal values as shown in the second column, given the fact that both expressions produce the same square root fractal value [1.731451582].

It is noteworthy then to point out that the cited values for the Planck Units [with one exception, Planck Charge] are produced by employing the expression for the speed of light in millimeters or kilometers, and *not* by the employment of the fractal values for scientific notation or in meters. If one employs the fractal numerical values for the speed of light in scientific notation [2.99792458] or in meters [299792458.0] then the values listed below in the second column obtain; and not the values generally cited in the literature of physics.

So, we are able to see that the Planck Units that contain a square root expression within the terms of their equations, given their cited values in the literature, were produced employing the speed of light numerical expressions with a fractal 1.731451582 root. And this obtains in spite of the fact that the cited value for the Planck Unit of length is expressed in meters. One would expect a consistency: if one is seeking an expression of unit

length in meters, then the fractal numerical of the speed of light would be using meters as well, and not kilometers or millimeters.

Planck Length is given as $1.616252(81) \times 10^{-35}$ **meters**

$$l_P = \sqrt{\frac{\hbar G}{c^3}}$$

Planck Length employing the **299792.458 km/sec** for the speed of light; a root value of **5.475330657** obtains:

$$\frac{1.054571628 \times 6.67428}{2.694400242E16} =$$

$$7.038506325 / 2.694400242 = 2.612272006$$

$$\text{Square root of } 2.612272006 = \mathbf{1.616252457} \text{ fractal}$$

Planck Length employing the **299792458.0 m/sec** for speed of light; a root value of **1.731451582** obtains:

$$\frac{1.054571628 \times 6.67428}{2.694400242E25} =$$

$$7.038506325 / 2.694400242 = 2.612272006$$

$$\text{Square root of } 2.612272006 = \mathbf{5.111039039} \text{ fractal}$$

Consider the variation in fractal numerical values for each one of the Planck Units that contains a square root expression within its terms, using either the 299792.458 kilometer or the 299792458.0 meter expressions.

Unit	Based on fractal 5.475330657 root	Based on fractal 1.731451582 root
Planck Unit	Using 299792.458 kilometers; 299792458000 millimeters	Using 2.99792458 scientific notation; 299 792 458 CODATA
Planck length:	1.616252 fractal	5.111039039 fractal
Planck mass:	2.17644 fractal	6.882499394 fractal
Planck time:	5.39124 fractal	1.704859113 fractal
Planck charge: <i>[Planck charge / CWJ</i>	<i>[Anomaly] 5.930796805 fractal]</i>	<u>1.875545870</u> fractal <i>[Anomaly]</i>
Planck temperature:	1.416785 fractal	7.159681157
Planck momentum:	6.52485 fractal	2.06332141
Planck energy:	1.9561 fractal	6.185681973
Planck angular frequency:	1.85487 fractal	5.865587324
Planck current:	<u>3.4789</u> anomaly	1.100119456
Planck voltage:	1.04295 fractal	3.669598012

Even though one may manipulate the decimal places of the terms of the equations, due to the square root expression within the formulae, two distinct fractal numerical values result for anyone of the equations. The question remains as to what is the theoretical basis for choosing one fractal numerical value over the other in each case.

In other words, even if it could be shown that the Planck Units are consistent in their mathematical computations, and employ meters in order to derive meters, one would have to ask how one resolves the question of the variation in square roots. By using square roots in the equations of physical constants, one necessarily opens the way for two distinct fractal numerical values. It is necessary, then, to explain why one answer should be accepted over the other. One would have to explain why by using variations in the fractal values for the speed of light [2.99792458, 299792.458, 299792458.0, etc.] only one answer is acceptable when the square roots of these fractal expressions produce two possible answers.

The root value of 5.475330657 was employed in the all of the Planck Units except that of Planck Charge, which evidently employed the root 1.731451582 value for c . In a sense, then, employing different decimal values for the speed of light in the equations subverts the very logic behind those equations as a set of constants. Employing different roots represents an inconsistency in the reasoning and produces an interruption in the logic of the Planck values thus denying any degree of comparability among them.

Hence, the Planck Unit for charge presents an inconsistency with regard to the other units. In the Planck Charge unit, the fractal values of root 1.731451582 for c are employed in order to derive the value given as 1.875545870, whereas in the values listed before it the root 5.475330657 for c was employed. In the case of the Planck Charge, to be consistent with the previous units, the 5.475330657 root value would have had to be employed for the Planck charge unit to then produce the 5.930796805 value so derived.

When the 5.475330657 root value for c is employed in the Planck *charge* equation, as in the previous formula, then a distinct fractal value is produced as in 5.930796805 fractal. In this manner, the use of distinct square root values is inconsistent with regard to the Planck Units. In some Planck Units the 1.731451582 root for c is used, while in others the 5.475330657 root value for c is employed.

Therefore, not only are there distinct possibilities in deriving the Planck Unit values, but there is no consistency between one Planck Unit with regard to the others as presented in the literature regarding the speed of light in its different fractal expressions.

With the Planck Unit for current something odd makes its appearance. When computing the value for Planck Current one does not obtain a distinct fractal numerical value by employing either the fractal root values of 1.731451582 and/or 5.475330657 for c , as occurred with the other Planck units. In fact, if one employs different fractal values for the **other terms** in some of the Planck formulae, as in for example the permittivity vacuum, **8.85418782**, **0.885418782**, then different values will obtain.

Consider the Planck Current as given in the literature:

$$\text{Planck Current } I_P = \frac{q_P}{t_P} = \sqrt{\frac{c^6 4\pi\epsilon_0}{G}} = 3.4789 \times 10^{25} \text{ A}$$

Computing:

the speed of light⁶ *times* 4 *times* pi *times* the permittivity of free space

$$\underline{299792.458^6} \text{ times } 4 \text{ times } 3.141592654 \text{ times } \underline{8.85418782} =$$

$$7.259792663 \text{ times } 12.56637061 \text{ times } 8.85418782 =$$

$$= 8.077608715$$

$$\text{divided by } G = 1.210259191 \text{ fractal}$$

$$\text{square root of fractal } 1.210259191 = \underline{1.100117808} \text{ fractal}$$

So, the cited **3.4789** value does not make its appearance. Then, I changed the decimal place for the speed of light:

speed of light⁶ *times* 4 *times* pi *times* permittivity of free space

$$\underline{299792458.0^6} \text{ times } 4 \text{ times } 3.141592654 \text{ times } \underline{8.85418782} =$$

$$7.259792663 \text{ times } 12.56637061 \text{ times } \underline{8.85418782} = 2.53765562$$

$$\text{divided by } G = 1.210259191 \text{ fractal}$$

$$\text{square root of fractal } 1.210259191 = \underline{1.100117808} \text{ fractal}$$

Again, even with changing the decimal fractal expression for the speed of light the cited value for the Planck Unit for current [fractal **3.4789**] does *not* appear. Now, even by employing either of the fractal roots for the speed of light the same value [1.100117808] derives from the computations, unlike what happened with the other Planck Units, where different results obtain.

It took me some time to discover that it mattered not whether the decimal place of the speed of light value was changed in the Planck Current formula, as in the other Planck Units. The trick in the Planck Current is to change the decimal place in the term of the permittivity of free space, that is, as in from **8.85418782** to **0.885418782**. Essentially, this means instead of using the fractal root expression **2.975598733** the fractal root **9.409669399** is employed. Consider the following, by changing the decimal place in the permittivity of free space the cited fractal numerical value [3.4789] makes its appearance as offered in the literature; again due to the square root factor in the equation, only this time with regard to the permittivity of free space and not as of *c*:

The permittivity of space as **0.885418782** [fractal root 9.409669399], *instead of* as **8.85418782** [fractal root 2.975598733]:

speed of light⁶ *times* 4 *times* pi *times* permittivity of free space

$$299792.458^6 \text{ times } 12.56637061 \text{ times } \mathbf{0.885418782} = 8.077608715$$

$$[\text{or, } 299792458.0^6 \times 12.56637061 \times \mathbf{0.885418782} = \mathbf{8.077608715}]$$

$$\text{divided by } G = 1.210259191 \text{ fractal}$$

$$\text{square root of fractal } 1.210259191 = \mathbf{3.478877968 \text{ fractal}}$$

Finally, with those changes, the Planck current value of fractal **3.4789** makes its appearance. Again, its appearance however is not determined by the fractal expression of the speed of light value in the equation, but rather by the fractal value of the permittivity of free space [8.85418782 and/or 0.885418782].

The previous computational exercises are important for comprehending the numerical fractal values proposed by the Planck Units.

Obviously, the counter-argument can be heard already that one must employ the permittivity of space with the actual number of decimal places cited for the permittivity of space. But, that argument does not hold for the variation due to the decimal place arrangements of the speed of light which are given in distinct forms throughout the literature, basically with a floating decimal place. And, further that counter-argument does not apply to the Planck Charge which as we observed reverses the logic of the value for the speed of light in its derivation.

In other words, even though one may obtain directly the 3.4789 value as proposed by the Plank Unit current, one would then have to explain the relevancy [or not] of the 1.100117808 fractal numerical expression accordingly. In a sense, one would have to question the very nature of scientific notation and the idea of significant numbers, in order to be able to explain that one value derived from square roots is significant while another value derived thereof is not. One would have to explain how the square root of 299792.458 is significant at times, and how the square root of 299792458.0 is insignificant, and/or vice versa. Such an explanatory task, in my mind, is impossible and without merit, other than as herein described. In other words, it is impossible to reject one square root and accept the other. Both exist, and each one requires an explanation regarding its nature.

In a sense, then, the explanation would necessarily revolve around the idea that the Planck Unit for mass is 2.17644 when the square root of the speed of light is employed as 299792.458 **kilometers**/sec and it is 6.8825 when the square root of the speed of light as 299792458.0 **meters**/sec is employed.

<i>Unit</i>	<i>299792.458-based</i>	<i>299792458.0-based</i>
	<i>Root 5.475330657</i>	<i>Root 1.731451582</i>
Planck mass:	2.17644 fractal	6.882499394 fractal

Essentially, the problem with the natural units, and especially with the Planck Units of measurement, is the fact that they contain square root expressions within their formulae. Such obtains given the different square root values for a specific term within the equations, as illustrated above by the speed of light in a vacuum.

I might add, that in reviewing many of the cited physical constants within physics and chemistry, one does not find a large number of equations

based on square roots. The Planck Units are an exception to this apparent rule. Whenever one utilizes a root value procedure in an equation in physics, the variation of answers is necessarily designed into the roots themselves. Any equation in physics based on square roots will by definition offer two possible fractal numerical values as answers.

To expect to have the same, single numerical expression for the Planck Unit of **length**, when employing two different fractal values for the speed of light as measured in millimeters, centimeters, meters or kilometers with different root values, is simply impossible ---mathematically speaking. One would expect the physicists to employ the 29959458.0 unit of meter in the computations of the Planck Units, but as shown the fractal value of **5.475330657** corresponding to 299792458000.0 **millimeters** and/or 299792.458 **kilometers** has been applied (with an exception or two).

With regard to the Planck Units, it is impossible then to propose equations of physics based on square roots and wish to derive a single answer as being *the* specified constant value. In a sense that is a contradiction of terms. All general equations that contain a square root expression will produce two possible answers depending upon the fractal numerical values employed in the terms of the equation. Three possible answers will derive for equations employing cube roots; four possible answers for powers of four; and so on.

The Planck Units are forwarded as though a single result [constant] exists without any consideration for the fractal root values of the speed of light within their computations, or for the other terms such as the permittivity of free space. One would have to present both, in fact all, fractal numerical values for each equation in order to eliminate the inconsistencies that occur with Planck charge and Planck current, ---not to mention with every single equation that contains a square-root expression within any key formula in physics.

Subsequently, there exists the need to discuss the very concept of a natural unit.

Part II

Theoretical Considerations Regarding Natural Units

The apparent lack of consistency in the Planck Units using square roots brings into question what theoretical physics means regarding the analysis of the properties of matter-energy. If the Planck Units are telling us that there is only one constant for each cited unit, while the math is telling us that there are two fractal numerical values, then what consequence does this have for understanding how matter-energy exists.

The Planck Units supposedly represent minimum **natural** units of measurement in theoretical physics, as in the minimum natural length *that may be measured*. In spite of the attempt to present the Planck Units as natural units of measurements, **anthropomorphic** units are being created. This occurs because anything within matter-energy that human beings wish to measure must be done based upon anthropomorphic, human-oriented arbitrary units. We humans are doing the measuring; the measuring is not done in Nature, in matter-energy. The ultimate question concerns the concept of measurement, not the basic units of matter-energy.

Given that the theoretical physicists are attempting to measure exactly how matter-energy behaves, one would expect them to employ terms and concepts that reflect matter-energy exactly.

The variation in the parametric concept [significant numbers per decimal place] of the measuring ruler influences the outcome of the numbers being generated. In using certain Planck formulae based on square roots, when one employs different decimal places in the computations, then different numerical values are produced. Even though one is employing a fractal expression of c , the speed of light in a vacuum, in the case of the majority of the Planck Unit formulae, they are employing the distinct fractal root of c values, producing distinct fractal numerical values for the constants. Such a distinction in the numerical value of the units has to do with the derivatives of square roots, as shown in Part I of this essay. So, if one uses the root value 1.731451582 for c , a different value obtains than if one uses the fractal root value 5.475330657 for c .

The distinction in the fractal numerical values of the constants resulting as of the equations is not due to the expression of c as such, but rather as of the distinct root values [1.731451582 and 5.475330657] for c . One looks at the level of different c -numbers, with a different decimal placement [2.99792458, 299792.458, 299792458.0, etc.], and one cannot imagine a distinction in the resulting values of the unit constants. But, when one considers the distinction in the **square-root values** of those numbers, then one may easily understand the variation in the results produced by those equations.

In other words, students look at the formulae of the Planck Units and considers the speed of light expressed in nanometers, millimeters, centimeters, meters, kilometers, and they do not suspect any variation in the answer, since one knows that due to the metric system a **floating decimal** causes no distinction in the unit measurement as such; they are mere fractal expressions of the same number/value. But, at the level of the computational math of the formulae, within the bowels of the equations, the different root values for those speed of light measurements do vary and they do produce opposing constant values [in Planck mass 1.616252 as against 0.5111039039].

If the Planck Unit for a particular aspect is given as a certain fractal numerical value, one would expect that numerical fractal value to be the same for all computations, but this is not the case. There are different numerical expressions for the Planck Units according to which fractal root value is employed in the terms of the formulae that produced that particular constant value.

In this sense, it is impossible to convert a particular fractal value of a specific Planck Unit into **scientific notation** [2.99792458] as though that numerical value remains the same for all computations for that Planck Unit. The scientific notation remains the same at the level of the numerical value itself in the term, but not at the level of the root values. As shown above, when employing **square roots**, *theoretically two numerical fractal values exist for each Planck Unit*. Converting one of those selected values into scientific notation as though it were the only solution to the equation offers an erroneous idea about that particular Planck Unit.

The square root value for the scientific notation figure, 2.99792458, is 1.731451582 for c . Intriguingly enough, although the constant values of the

Planck Units are expressed in scientific notation, they do not use that root value, but rather they employ the root value of 5.475330657.

For example, Planck Length yields two specific values:

Unit	Using root 5.475330657	Using root 1.731451582
Planck length:	1.616252 fractal <i>[given]</i>	5.111039039 fractal <i>[alternate]</i>

Now, to take the 1.616252 value and present that one in scientific notation as though that is the Planck Unit for all combinations of the fractal numerical speed of light is incorrect. One could easily have chosen the **5.11103039** value to express the same unit value. And, like so for the other Planck Units based on terms of square roots as illustrated above.

The inconsistency of the procedure became obvious as the Planck Unit for charge was analyzed:

Unit	Based on 5.475330657 root	Based on 1.731451582 root
Planck charge:	<i>[Inconsistency]</i>	1.875545870 fractal
<i>[Planck charge / CWJ</i>	<i>5.930796805 fractal]</i>	-

Following the procedural logic of the other Planck Units, the Planck charge should have been 5.930796805, but is listed as constant value 1.875545870.

The Planck charge presents the fractal numerical value 1.875545870 value in scientific notation [again, as though this were the only one available]. But, as we demonstrated the 1.875545870 value is derived by employing the speed of light as 299792458.0 m/s, while most of the other Planck Units are derived by employing the 299792.458 kms/sec value. If one were to be consistent in the Planck Units by employing the 299792.458 km/sec value for the Planck charge, than its value would be fractal 5.930796805 as demonstrated.

The inconsistencies in the Planck Units is the employment of fractal numerical values for the speed of light and/or for the permittivity of free

space [8.85418782; .885418782] based on distinct fractal values for those constants. Hence, distinct fractal values employed in the terms of the Planck equations/formulae produce distinct numerical values for the Planck Units.

The conclusions from such computations are diverse. The Planck Units are not minimal, even within anthropomorphic related units of measurement as at times the minimal expression of certain terms are not minimal themselves. It would be logically impossible to produce minimal expressions of measurement from terms that are themselves multiples thereof.

Therefore, given the nature of the formulae that contain an expression of the square root of one or more of the terms, two different fractal numerical values will be produced by such computations. The Planck Units have been presented with only one supposedly numerical product, which according to the nature of square roots represents at least half the results possible, or just incorrect.

Said observations mean that all of the theoretical work based on the Planck Units cited that contain square root expressions is openly partial and possibly deficient.

Other systems of natural units exist. One would suspect that those systems, inasmuch as they contain square roots also succumb to the same kind of deficiency and partiality in their results as in the cited Planck Units. Some systems of natural units that require examination are the **Stoney Units** and the **Schrödinger Units**, both of which also contain some equations in square roots.

Stoney Units

Quantity	Expression	Metric Value
Length (L)	$l_S = \sqrt{\frac{Ge^2}{c^4(4\pi\epsilon_0)}}$	1.38068×10 ⁻³⁶ m
Mass (M)	$m_S = \sqrt{\frac{e^2}{G(4\pi\epsilon_0)}}$	1.85921×10 ⁻⁹ kg
Time (T)	$t_S = \sqrt{\frac{Ge^2}{c^6(4\pi\epsilon_0)}}$	4.60544×10 ⁻⁴⁵ s
Electric charge (Q)	$q_S = e$	1.60218×10 ⁻¹⁹ C
Temperature (Θ)	$T_S = \sqrt{\frac{c^4e^2}{G(4\pi\epsilon_0)k^2}}$	1.21028×10 ³¹ K

The Stoney Units that are based on square root computations in their terms also pose two different fractal numerical results inasmuch as any one of their terms may employ a distinct decimal placement, as in the roots of the speed of light [either 299792.458 kilometers/second or 299792458 meters/second]. The alternate fractal numerical results would be as follow for those equations:

<i>Unit</i>	<i>Metric Value</i>	<i>Alternate Value</i>
Length	1.368068	4.366093519
Mass	1.85921	5.87939047
Time	4.60544	1.456368003
Temperature	1.21028	3.827241406

In order to check for alternate fractal values for the results of the equations in physics, in the case of Stoney Units, one may square the metric value given, then move the decimal place of that result one placement in either direction to the left or right, and then find the square root of that value.

Schrödinger Units

Quantity	Expression	Metric Value
Length (L)	$l_{\psi} = \sqrt{\frac{\hbar^4 G (4\pi\epsilon_0)^3}{e^6}}$	2.59276×10 ⁻³² m
Mass (M)	$m_{\psi} = \sqrt{\frac{e^2}{G(4\pi\epsilon_0)}}$	1.85921×10 ⁻⁹ kg
Time (T)	$t_{\psi} = \sqrt{\frac{\hbar^6 G (4\pi\epsilon_0)^5}{e^{10}}}$	1.18516×10 ⁻³⁸ s
Electric charge (Q)	$q_{\psi} = e$	1.602176487×10 ⁻¹⁹ C
Temperature (Θ)	$T_{\psi} = \sqrt{\frac{e^{10}}{\hbar^4 (4\pi\epsilon_0)^5 G k^2}}$	6.44490×10 ²⁶ K

With regard to the Schrödinger Units, one may note that they do not obviously contain the term *c*, the speed of light in a vacuum. However, within the term of the Permittivity of Free Space [i.e., electric constant or vacuum permittivity], the speed of light plays a part in that particular computation. The vacuum permittivity results from the reciprocal of the speed of light squared times the magnetic constant. So, even though the speed of light is not apparent in the Schrödinger equations it does exist buried within its terms and therefore influences the decimal placement issue within square roots.

For that reason, a variation in the decimal placement of the vacuum permittivity [as in 8.85418782 and 0.885418782] will influence the outcome of the fractal numerical value for the equation within which it is contained.

In other words since the employment of the speed of light in meters or kilometers and its corresponding decimal placement is contained within the vacuum permittivity value, then one can derive either alternate values for the terms expressed in square roots for said equations.

Again, one must note that the majority of the constants or constant-like values [inasmuch as the permittivity of free space is not a constant value], are expressed in terms of **meters** [as in Farad per **meter** or equivalently as $A \ s \ kg \ m$]. Such occurs throughout the constants in that their expression is in meters. Yet, in the Planck Units the speed of light is employed as decimal placement corresponding to **millimeters and kilometers** which reflect a root value of 5.475330657. And, as we have seen, if one employs the decimal placement as in **meters** [with a root value of 1.76314501582] for that term, then an alternate fractal numerical constant will obtain completely different from the one offered in the literature.

Given this now confirmed fact through the computations presented in this essay, one may suspect that the theoretically abstracted natural units are deficient and inconsistent in their mathematical results. Mainly, this is due to the fact that in general the root expression for c has been employed as the decimal amount of kilometer per second, instead of the root expression for meters. Had the Planck constants with a square root term in them employed the expression for c in meters, then their fractal numerical results would be distinct as indicated in this study.

The fact, however, that the majority of the cited Planck units with square roots employed expressions in millimeters and kilometers and only one employed the expressions in meters represents an obvious inconsistency in their fractal numerical derivations.

The Stoney Units and the Schrödinger Units, the so-called natural units, also contain square root expressions and also succumb to similar inconsistencies inasmuch as they involve the speed of light in a vacuum and the possibility of effecting the computations in either meters or kilometers.

Now, given the fact that the Schrödinger Units contain the speed of light hidden within the term of the permittivity of free space, without listing the speed of light visibly in the equations, causes the determination of which fractal numerical result is due to which particular decimal placement expression for c .

I have listed above the given metric values for the Stoney and Schrödinger Units as offered in the literature, along with the alternate values. However, it would require much research to decide which particular fractal numerical value is derived from a computation involving the speed of light in meters or in kilometers to unravel their meaning. For now, I have simply taken as given the metric values listed in the literature and simply placed in a parallel manner the obvious alternate values. Again, one could change any one of the decimal placements within any of the terms of the equations and derive different relationships for primary/secondary root values as of the square root computations in the equations.

From the above, it becomes apparent that much of the work done regarding the key equations in physics requires a detailed analysis and review. Once the inconsistency of employing root values in the equations that are stated in terms of meters, and the fact that different fractal numerical values obtain thereof, then one would have to examine the entire history of the key equations in physics in order to understand which values derive from which.

For even though, a particular unit may have its result expressed in meters, as in the Planck Unit length, the fact that a decimal placement for millimeters and kilometers was employed in the terms of the equation, would render a fractal numerical value that does not correspond to meters, but rather kilometers.

In another sense, then, the only Planck Unit, that contains a square root expression in its terms, and that is expressed in meters is that of the Planck charge. The other Planck Units cited here have their computational terms expressed in kilometers, but their results expressed in meters. These latter Planck units should have as their fractal numerical expression the alternate values cited in this study and not the ones given generally in the literature.

To be consistent then, the alternate Planck Units should also be expressed as:

Unit	Using 1.731451582 root value
Planck Unit	Using 299 792 458. 0 meters or, 2.99792458 scientific notation, CODATA expression
Planck length:	5.111039039 fractal
Planck mass:	6.882499394 fractal
Planck time:	1.704859113 fractal
Planck charge:	1.875545870 fractal [Same]
Planck temperature:	7.159681157
Planck momentum:	2.06332141
Planck energy:	6.185681973
Planck angular frequency:	5.865587324
Planck current:	1.100119456 [Anomaly]
Planck voltage:	3.669598012

From this analysis, it becomes apparent that in computing the majority of the cited fractal numerical values for the Planck Units, neither the expression of the speed of light in **meters** [299792458] nor its expression in the **scientific notation** [2.99792458] was employed. In fact, it is now obvious that the fractal numerical values of the cited Planck Units were computed as of the expression for the speed of light in **millimeters and kilometers**.

Certainly, physicists who might wish to defend the inconsistencies would state that we are dealing with the metric system and its matters not which particular metric division is employed, either meters or kilometers. One of the most powerful tools of the 10-base metric system is the floating decimal place. And, that would hold were it not for the fact that the key equations in physics cited here have within their terms the square root of the speed of light. And, we have indicated depending upon the decimal place is allowed to float produces distinct fractal numerical values in the equations. Square roots trump floating decimal places in the terms stated.

Further, when one considers how many equations in theoretical physics exist that contain an expression based on the square root of one or more of its terms [and even more so with cube roots and above] one can only wonder how many of the results from those equations in physics might suffer from some of the inconsistencies illustrated here regarding the Planck units. Some of the more significant equations in physics come to mind: the wave equations in matter; the Lorenz transformation; general relativity; plasma physics; the induced dipole interaction; among various others.

All **natural units of measurement** are man-made, anthropomorphic or humanly derived. No natural units of measurement actually exist. What does exist in spacetime, in matter-energy, are natural units of proportion. As in, for example, the proportion of the diameter of a circle to its circumference [π]. There is no base unit in Nature in as much as all forms of matter-energy, in being able to be combined, produce minimum/least, and maximum/most forms of matter-energy. To choose a supposed minimum expression of matter-energy, or even worse, of spacetime/movement is a theoretically abstracted task set out by human beings.

Human beings are intrigued by what makes up reality, spacetime, matter-energy and thus seek to find the inner workings of the atom, only to find the proton, neutron and electron. Then, they go on to find that even those so-called minimal manifestations have yet other inner workings [quarks, leptons, etc.]. Yet, even then, as human beings approach the unapproachable reality and are faced with manifestations of pure energy without mass, they continue to seek that supposed explanatory datum of how reality exists; enter the Higgs-Boson particle. And, then they find out that the Higgs Boson particle may weigh more than the other sub-particles from which the supposedly sub-atomic elements came and are thus confounded by finding a larger particle as the origin of the smaller ones.

Nature does not measure; it just exists/happens. Rings on a tree, segments of a tape-worm, so many apparent units exist in Nature, but with variation. One knows the reasons, to an extent, as to why these apparently **natural units of existence** occur and show variation in their size. The rings of trees are measured by human beings. But one would not expect to derive the minimal unit of a ring of a tree as a constant. Yet, when one theorizes about another level of matter-energy, that of spacetime and matter-energy in general at the Cosmos level, physicists begin toying with the idea of a minimal unit of measurement, of those natural units of existence even though they understand quite well the term of motion and change, not to speak of relativity and quantum leaps per their own definitions.

The theoretical physicists set about “normalizing” their units to the value of 1.0 in hopes of discovering a unit of measurement for all space, all time, all movement.

One already knows the relationship of the diameter to a circle: pi as constant value 3.1415926254..., which is unending as far as we now know. But this unending number should tell us something about Nature. One may assign the value of 1.0 to the diameter and then know that the circumference of that unit circle is 3.1415926254 times that length. And, even though the numerical expression of the circle is apparently not an enclosed number, the circle is an enclosed/cut-off entity at some point. The circumference of a circle is not an unending line, as is the unending numerical expression of pi.

One may assign the value of 1.0 to the circumference and know that the diameter is 0.318309886 the length of that circumference. One may assign the number 360 to the circumference [as in a 360-degree circle], and know that the diameter measures 114.591559....; and so on. Any spacetime/motion event, any form of matter-energy may be assigned the unit value of 1.0 in a theoretically abstracted analysis, as in physics or whatever human endeavor. But, that does not mean that there exists a unit 1.0 diameter, a unit 1.0 circumference, or a unit 1.0 whatever for any form of matter-energy.

The relational and proportional aspect of matter-energy goes beyond any theoretical abstraction that we humans may make of reality, of spacetime.

One could assign the proportion of pi [the relation of a diameter to a circle's circumference] as a unit of measurement. In that sense, pi is an expression of a dimensionless number. Any assignment of 1.0 to a unit-diameter or to a unit-circumference is merely **dimensionful**, as the length of a diameter and the length of a circumference may [and do] vary. Yet, the proportion of length of the pi measurement is unchanging and constant, the same for every circle. And, yet, the circumference of any circle is itself the exact expression of pi for that circle. Think about it; the length of the circumference is pi for that circle.

Pi-proportion is a natural unit. Yet, pi is employed in some of the Planck Units as though it supports the so-called natural units that are trying to be expressed as the result of the Planck formulae.

In this sense, the proportion of pi is dimensionless as it never alters throughout spacetime/motion. Now, its absolute value varies as do the diameter and circumference of the circle vary, but its proportion remains always the same. And, as a symbolic representation of proportion, pi is thus incommensurable outside of a particular circle [circumference].

Only when the diameter of a circle is set to unit 1.0 is the circumference of that circle a physical measurement of pi; and then it is simply the measurement of that circle's circumference; measuring it is the circumference, nothing more. Hence, pi is physically always visible on any drawn circle. What is difficult to imagine is the proportion between that diameter and its circumference, no matter how long and hard one looks at a circle with its diameter drawn on it.

So, one has the circumference of a circle as an absolute spacetime/motion measurement of length on a given circle. Then one has the circumference as a physical representation of pi, the proportion of the diameter to the circumference itself.

In a sense, then, proportion is constant in matter-energy, as a relationship of different spacetime events, as of different forms of matter-energy. In a sense, that is what the Planck Units are attempting to accomplish; a measurement of proportion. But, as with pi, there is no physical measurement of proportion as such. The difference with some of the Planck Units is that instead of producing a single numerical value for the representation of the proportional value cited (as in pi), they are actually

deriving two possible numerical value outcomes from their formation of square roots, but they may recognize a single answer. For whatever reason, the Planck Units present only a single numerical value as the selected option, possibly to reinforce the idea of a physical constant.

In the Planck Units based on square roots, there is no single option; there are two optional root values and two outcomes possible.

For example, the consider the square root optional values for pi:

$$\text{Square root of fractal pi } 0.3141592654 = 0.5604991217$$

$$\text{Square root of pi as } 3.141592654 = 1.772453851$$

The square root of pi is 1.772453851. But, the square root of fractal value pi is 0.5604991217. Both are correct. Yet, both produce distinct fractal numerical values within specified equations which in themselves are correct with distinct meaning according to their fractal values and the equation within which they finds themselves. And it occurs with the speed of light, the permittivity of free space, etc., in the Planck Unit equations.

In a sense, then, there is no minimum unit of measurement of spacetime or of matter-energy as in minimum length. The minimum length possible is zero; obviously. Or, the minimum is the smallest gradient above zero that one may theoretically envision, which is according to the measuring system devised by human beings. That could be the fractal value decimal point 43-something as derived in the Planck Unit for time, after the Big Bang, or whatever other chosen unit of measurement one may wish to surmise.

But, remember, any minimum unit value assigned to a particular aspect of spacetime [mass, length, time] is simply that: an assigned value. As in the case of pi, 3.141592654, it may be an assigned value to a proportion and in that sense is apparently natural. But, nonetheless, even the 3.141592654 value is an assigned value; one assigned to the proportion of a diameter | circumference relationship by human beings on this planet. Just as we have assigned that proportion 1 | 3.141592654, we could assign those values as 756 | 2375.044046, and so on.

For any Planck Unit value one would then understand that those so-called natural units would be defined as one gradient value above zero, no matter which numbers are actually being assigned. To choose any other value above zero+, would be an arbitrary assignment, meaning marking off an arbitrary among of mass, length, time, or whatever other aspect of that which is being measured.

An analogy would be like asking what is the smallest measurable circle. Take it from there. What is the smallest commensurable diameter of a circle; the smallest value for pi, the smallest sphere, etc. The answer should be obvious: the smallest value for pi is either 1.0 as absolute unit 1.0, or 3.141592654 as absolute proportion. In either case, and in any case, the smallest possible constant pi relation is either one, inasmuch as the circle/sphere is zero+ [however minimally one may design the plus sign --- either show it as unit 1.0 or place an infinite number of zeros before it].

Other than that, for any form of matter-energy, if there is a space, then any zero+ space is minimum for space. One has no space, which is non-existent; then, one has the next space+ which is the minimum above that zero-space.

Measuring any of these events constitutes a human activity, and, thus reflects any *chosen* amount of spacetime/motion or matter-energy [mass, length, time]. The concept of a minimally functional spacetime is defined by human beings. Spacetime simply functions [or not]; it exists or does not exist. Measure the empty space in front of you as you sit reading this. What number do you obtain when looking at empty space? A nano-measure of empty space becomes meaningless unless related [proportionally] to a specific matter-energy event [even if that be the dark elusive matter/energy].

Yet, for spacetime/motion that apparently empty space is functional to the presence/absence of matter-energy. In fact, the concept of empty space has been done away with as one perceives that even the empty Cosmos is full of dark matter/energy. Key to our understanding this line of reasoning are the concepts of measurement [a human endeavor] and the concept of what is constant [for all space and all time]. As was so vigorously repeated throughout the nineteenth century, the only constant is change itself; a contradiction and precision of terms.

When everything [every spacetime event, every form of matter-energy] is in flux, moving, in motion, then how can one possibly identify physical and chemical constants. The only way is with regard to relationships of proportion, of chosen events in relation to one another, as shown regarding that between the diameter and circumference of a circle.

Even a rock that appears to be static is bursting with energy, as in the making of a flint arrowhead that lets off sparks of energy when flaked. Time is movement; mass is movement. No matter which spacetime/motion event one ponders, the first thing to identify is its internal/external, its complete movement.

The circumference of a circle as proportion to its diameter is constant. The circumference of a circle as absolute length for that circle is inconstant, varying as we draw the circle larger/smaller ---yet, the pi-proportion has remained constant [if we draw a perfect circle of course].

In a sense then, to seek a constant natural unit of mass, length, time and the like, as in the Planck Units, is a vanishing task. Spacetime/motion, the forms of matter-energy, do not remain motionless enough for us to achieve such a task. In fact, the very spacetime definitions of terms such as mass, matter, length, time, energy are themselves constant motion/movement, in constant change. And, the only constancy available for human measurement pertains to their aspects of proportionality among those spacetime/motion, matter-energy events observed.

The concept that requires examination is that of the idea of a unit. A unit is a perceived and assigned division which the human being has arbitrarily forced upon spacetime/motion, and its forms of matter-energy. In a relational analysis of spacetime any matter-energy event may be perceived as and assigned the concept of unit [1.0]. And, thereof all other matter-energy events related to that assigned event may be compared and analyzed.

There are no natural units as such, in the terms that Planck Units seek to derive them from numbers or, into specified numerical values. This becomes even more obvious as we consider the deficient nature of the square-root formulae in the Planck Units, where there are two numerical answers to each Planck Unit ---even though historically only one single answer is offered as being the unit-value for that particular constant.

It must be emphasized, however, that there appear to be constant relations of proportion among different/specific spacetime events, specific forms of matter-energy. These relationships of proportion may be assigned a particular numerical value and then called a unit-value. But, even then, one must exercise caution. I have been showing how many relationships within matter-energy reveal a proportional numerical value range of 1.366 - 1.38. [www.earthmatrix.com/sciencetoday/inverse_fine_structure_constant.html]. But, even then, one must take care as to what those values reflect in proportions of spacetime/motions relationships of matter-energy.

In considering the Planck Units, one would have to know whether all of the numerical value cited in the different Planck Units reflect the same scale of 0-infinity. Obviously, they do not. The only scale of comparative relevancy concerns that of proportion, and then we are speaking about the ratios of the terms. But, in order to relate the proportional relationships of different spacetime/motion, or matter-energy events, one would require yet another essay of equal or greater length than this one. Such a discussion, then, would take on an explanation of what the different values within square-root computations mean; what the three values within cube-root computations would mean; and what the four values within roots of the fourth power computations would mean; and so on, in terms of units of assigned measurements. That discussion represents another line of reasoning about root values. For now, the theme of the natural units of measurement have been discussed ever so tentatively.

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P.O. Box 231126, New Orleans, Louisiana, USA 70183-1126

*www.earthmatrix.com www.theschemata.com www.paleoanimation.us
johnson@earthmatrix.com*

Addendum

Planck Units

Source: *www.wikipedia.com*

Table 1: Fundamental physical constants^{5.930996805}

Constant	Symbol	Dimension	Value in SI units with <u>uncertainties</u> ^[1]
<u>Speed of light in vacuum</u>	c	$L T^{-1}$	299 792 458 <u>m/s</u>
<u>Gravitational constant</u>	G	$L^3 M^{-1} T^{-2}$	$6.674\ 28(67) \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
<u>reduced Planck's constant</u>	$\hbar = \frac{h}{2\pi}$ where h is <u>Planck's constant</u>	$L^2 M T^{-1}$	$1.054\ 571\ 628(53) \times 10^{-34} \text{ J s}$
<u>Coulomb force constant</u>	$\frac{1}{4\pi\epsilon_0}$ where ϵ_0 is the <u>permittivity of free space</u>	$L^3 M T^{-2} Q^{-2}$	$8\ 987\ 551\ 787.368\ 1764 \text{ N m}^2 \text{ C}^{-2}$
<u>Boltzmann constant</u>	k	$L^2 M T^{-2} \Theta^{-1}$	$1.380\ 6504(24) \times 10^{-23} \text{ J K}^{-1}$

Key: L = length, T = time, M = mass, Q = electric charge, Θ = temperature. The values given without uncertainties are exact due to the definitions of the metre and the ampere.

Table 2: Base Planck units

Name	Dimension	Expressions	SI equivalent with uncertainties ^[1]	Other equivalent
<u>Planck length</u>	<u>Length</u> (L)	$l_P = \sqrt{\frac{\hbar G}{c^3}}$	$1.616\,252(81) \times 10^{-35} \text{ m}$	
<u>Planck mass</u>	<u>Mass</u> (M)	$m_P = \sqrt{\frac{\hbar c}{G}}$	$2.176\,44(11) \times 10^{-8} \text{ kg}$	$1.220\,862(61) \times 10^{19} \text{ GeV}/c^2$
<u>Planck time</u>	<u>Time</u> (T)	$t_P = \frac{l_P}{c} = \frac{\hbar}{m_P c^2} = \sqrt{\frac{\hbar G}{c^5}}$	$5.391\,24(27) \times 10^{-44} \text{ s}$	
<u>Planck charge</u>	<u>Electric charge</u> (Q)	$q_P = m_P 2\pi \sqrt{G\epsilon_0} = \sqrt{\hbar c 4\pi\epsilon_0}$	$1.875\,545\,870(47) \times 10^{-18} \text{ C}$	$11.706\,237\,6398(40) \text{ e}$

<u>Planck temperature</u>	Temperature (Θ)	$T_P = \frac{m_P c^2}{k} = \sqrt{\frac{\hbar c^5}{G k^2}}$	$1.416\,785(71) \times 10^{32} \text{ K}$	
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Table 3: Derived Planck units

Name	Dimensions	Expression	Approximate <u>SI</u> equivalent
<u>Planck area</u>	Area (L^2)	$l_P^2 = \frac{\hbar G}{c^3}$	$2.61223 \times 10^{-70} \text{ m}^2$
<u>Planck volume</u>	Volume (L^3)	$l_P^3 = \left(\frac{\hbar G}{c^3}\right)^{\frac{3}{2}}$	$4.22419 \times 10^{-105} \text{ m}^3$
<u>Planck momentum</u>	Momentum (LMT^{-1})	$m_P c = \frac{\hbar}{l_P} = \sqrt{\frac{\hbar c^3}{G}}$	6.52485 kg m/s
<u>Planck energy</u>	Energy (L^2MT^{-2})	$E_P = m_P c^2 = \frac{\hbar}{t_P} = \sqrt{\frac{\hbar c^5}{G}}$	$1.9561 \times 10^9 \text{ J}$

<u>Planck force</u>	<u>Force</u> (LMT ⁻²)	$F_P = \frac{E_P}{l_P} = \frac{\hbar}{l_P t_P} = \frac{c^4}{G}$	$1.21027 \times 10^{44} \text{ N}$
<u>Planck power</u>	<u>Power</u> (L ² MT ⁻³)	$P_P = \frac{E_P}{t_P} = \frac{\hbar}{t_P^2} = \frac{c^5}{G}$	$3.62831 \times 10^{52} \text{ W}$
<u>Planck density</u>	<u>Density</u> (L ⁻³ M)	$\rho_P = \frac{m_P}{l_P^3} = \frac{\hbar t_P}{l_P^5} = \frac{c^5}{\hbar G^2}$	$5.15500 \times 10^{96} \text{ kg/m}^3$
<u>Planck angular frequency</u>	<u>Frequency</u> (T ⁻¹)	$\omega_P = \frac{1}{t_P} = \sqrt{\frac{c^5}{\hbar G}}$	$1.85487 \times 10^{43} \text{ s}^{-1}$
<u>Planck pressure</u>	<u>Pressure</u> (L ⁻¹ MT ⁻²)	$p_P = \frac{F_P}{l_P^2} = \frac{\hbar}{l_P^3 t_P} = \frac{c^7}{\hbar G^2}$	$4.63309 \times 10^{113} \text{ Pa}$
<u>Planck current</u>	<u>Electric current</u> (QT ⁻¹)	$I_P = \frac{q_P}{t_P} = \sqrt{\frac{c^6 4\pi\epsilon_0}{G}}$	$3.4789 \times 10^{25} \text{ A}$

<u>Planck voltage</u>	<u>Voltage</u> ($L^2MT^{-2}Q^{-1}$)	$V_P = \frac{E_P}{q_P} = \frac{\hbar}{t_P q_P} = \sqrt{\frac{c^4}{G4\pi\epsilon_0}}$	$1.04295 \times 10^{27} \text{ V}$
<u>Planck impedance</u>	<u>Resistance</u> ($L^2MT^{-1}Q^{-2}$)	$Z_P = \frac{V_P}{I_P} = \frac{\hbar}{q_P^2} = \frac{1}{4\pi\epsilon_0 c} = \frac{Z_0}{4\pi}$	$29.9792458 \ \underline{\Omega}$

Table 4: How Planck units simplify the key equations of physics

	Usual form	Nondimensionalized form
<u>Newton's law of universal gravitation</u>	$F = -G \frac{m_1 m_2}{r^2}$	$F = -\frac{m_1 m_2}{r^2}$
<u>Einstein field equations in general relativity</u>	$G_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$	$G_{\mu\nu} = 8\pi T_{\mu\nu}$
<u>Mass–energy equivalence in special relativity</u>	$E = mc^2$	$E = m$
<u>Thermal energy per particle per degree of freedom</u>	$E = \frac{1}{2} kT$	$E = \frac{1}{2} T$
<u>Boltzmann's principle for entropy</u>	$S = k \ln W$	$S = \ln W$
<u>Planck's relation for energy and angular frequency</u>	$E = \hbar\omega$	$E = \omega$

<u>Schrödinger's equation</u>	$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) = i\hbar\dot{\psi}(\mathbf{r},t)$	$-\frac{1}{2m}\nabla^2\psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t) = i\dot{\psi}(\mathbf{r},t)$
<u>Hamiltonian form of Schrödinger's equation</u>	$H \psi_t\rangle = i\hbar\partial \psi_t\rangle/\partial t$	$H \psi_t\rangle = i\partial \psi_t\rangle/\partial t$
<u>Covariant form of the Dirac equation</u>	$(\hbar\gamma^\mu\partial_\mu - imc)\psi = 0$	$(\gamma^\mu\partial_\mu - im)\psi = 0$
<u>Coulomb's law</u>	$F = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r^2}$	$F = \frac{q_1q_2}{r^2}$
<u>Maxwell's equations</u>	$\nabla \cdot \mathbf{E} = \rho/\epsilon_0$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\nabla \times \mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial \mathbf{E}}{\partial t}$	$\nabla \cdot \mathbf{E} = 4\pi\rho$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\nabla \times \mathbf{B} = 4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t}$

Table 5: Today's universe in Planck units

Feature of present-day universe	Approximate number of Planck units
<u>Age</u>	$8.0 \times 10^{60} t_p$ (4.3×10^{17} seconds)
<u>Diameter</u>	$5.4 \times 10^{61} l_p$ (8.7×10^{26} meters)
<u>Mass</u>	Roughly $10^{60} m_p$ (3×10^{52} kilograms, only counting stars); <u>10^{80} protons</u> (sometimes known as the <u>Eddington number</u>)
<u>Temperature</u>	$1.9 \times 10^{-32} T_p$ (temperature of the <u>cosmic microwave background radiation</u> , 2.725 kelvins)