An Angular Perspective of Leonardo da Vinci's Vitruvian Man
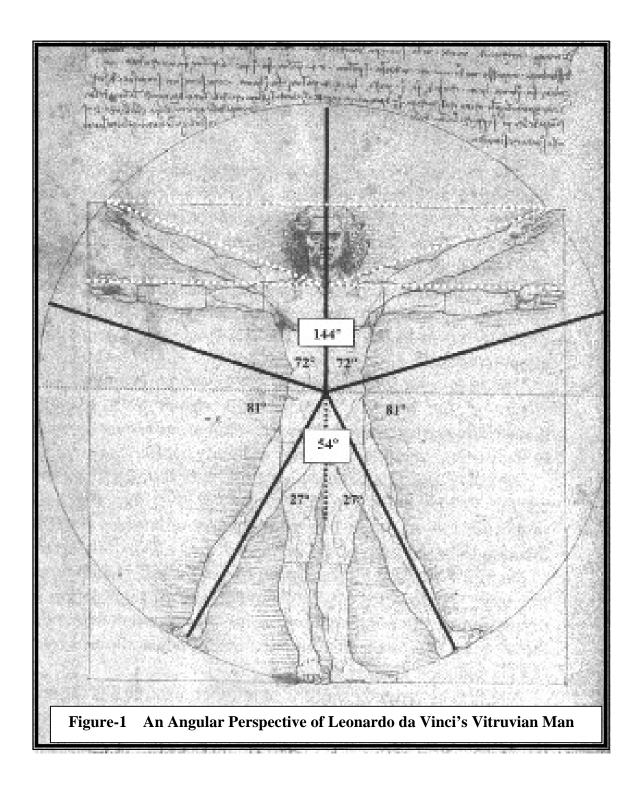
Utilizing the Phi (φ) Function Identity Rule of Natural Symmetry.

The famous drawing of the *Vitruvian Man* visually defines the nature of man's physical proportions in terms of the geometric bounds of the square and the circle (Figure-1). This dual projection of man into the overlaid square and circle may additionally provide the viewer with a sense of motion.

The projection into the square with his arms out-stretched horizontally, touching the sides of the square, and his legs together relays to the viewer the fact that the man's arm-span is equal to his height.

The man's projection into the circle, which is tangent to the square at the baseline, provides the reader with a pair of historically significant, *numerical mirror image* angles adjacent to the vertical line drawn through the body center. i.e. 72° & 27°.

Aware of the abundance of writings over the years that dwell on the Golden ratio Phi (φ) = (*1.618034···*) and the bodily proportions available in this drawing, the author has elected to mention only the ratio of the *vertical distance from the baseline to the naval, divided by the distance from the naval to the top of the head.* Ratio ≈ Phi (φ). The remainder of this section is a discussion of the angles formed by the arms and legs and their trigonometric values that may be expressed in terms of Phi as allowed by the Phi (φ) function identity rule. The first major historically significant number discussed is the angle extended by the outstretched arms over the head of the man in the circle. Contact is made with the fingertips at the edge of the circle and the angle is 144 degrees.

Cosine 144 = -0.8090 = -φ/2.

Figure-1   An Angular Perspective of Leonardo da Vinci's Vitruvian Man

The second historically significant number is the angle 54 degree that is extended by the open leg stance of the Vitruvian man in the circle. Applying the Phi function identity rule to this number, e.g. we divide 54 by 360 and obtain 0.15 parts of a revolution. The two decimal places signify that it is a Sine function of Phi. i.e. Sine 54° = 0.8090 = $\varphi/2$.

Note at this point that the half-angles of 144°and 54° are respectively 72°and 27°, a unique *numerical mirror image* pair that we will further examine. As discussed earlier, these historically significant numbers may from time to time take on other units from the Imperial system of units.

❑ The product of 72 and 27 is 1944. This historically significant number is the perimeter (in feet) of the largest semi-circle that can lie inside the base area of the Great pyramid. The arc length of the semi-circle is 1188[*] feet.   When this is added to the pyramid's width of 756 feet, it provides the perimeter of the half circular area.   i.e.
   1188 ft. + 756 ft. = 1944 feet.   Cosine 1944 = -0.8090···= -$\varphi/2$
   Also, the ratio 1188 / 756 = 11/ 7 = π_p / 2 = Pyramid Pi /2.

❑ The angle 72° and its adjacent angle 81° have a sum of 153°.
   Historically, number 153 relates to the quantity of fishes caught in the net of the disciple Simon Peter in the gospel of St. John.   The Cosine of 72° = 0.3090··· = *1/(2φ).*

   Note, the 153rd course of the Great pyramid is at an average height of 4379.85 inch[* *] = 365 feet.   It is often cited in reference to the number of days in a year. i.e. 365 d/y.

❑ The two 27°angles and their adjacent 81°angles have a total sum of 216°. The ratio presented by the arms of the Vitruvian man with the division of the circle into two parts is 144 : 216.

[*] *This historically significant of number (**1188**) is also the total digit sum of Table-1.*
[* *] *W.M.F.Petrie - The Pyramids and Temples of Gizeh – Course Data – Published London 1883.*

The ratio 144:216 = 2:3 is important here, in that Socrates, in a discussion of musical harmony in Plato's *The Marriage Allegory (Republic)*[8] comments that the "*human male*", prime number five, enters harmonic theory as an *arithmetic mean* within the perfect fifth of 2:3 – expanded to 4:5:6 to avoid fractions.

Did Leonardo di Vinci select the angular ratio 2:3 for placement of his Vitruvian "human male" in the circle, or was it just an unavoidable fact that was by nature the only possible choice? It is the author's belief that the angular perspective presented here, concerning the use of angles whose trigonometric functions can be expressed in terms of Phi (φ), offers further evidence of nature's influence on the great works of man.

Many angular images found in nature satisfy the Phi (φ) function identity rule, and such angles are found in many of man's creative works. These angles may occur, unknown to the artist, because of physical restrictions that nature places on the artist's subject. Just such a limitation is displayed by the angles required for the Vitruvian man to make four-point contact with his circle, when his navel is considered the focus of that circle. The viewer may also be unaware that this limitation is possibly a result of a biological Phi function requirement, unless they are told of it.

Phi Function Identity Rule

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1. If (n) is an integer divisible by 9, and (n) ÷ 360 contains one decimal place, (i.e., .1, .2, .3, .4, .6, .7, .8, .9), excluding (.0 & .5), then the Cosine (n) can be expressed as a function of Phi. ---- i.e. cosine (n) = $f(φ)$.
If (n) ÷ 360 ends with (.0 or .5), then Cosine (n) = ± 1.

2. If (n) ÷ 360 has two decimal places which is an odd multiple of (.05), (i.e., .05, .15, .35, .45, .55, .65, .85, .95) excluding (.25 & .75), then the Sine (n) can be expressed as a function of Phi. --- i.e. sine (n) = $f(φ)$.
If (n) ÷ 360 ends with (.25 or .75), then Sine (n) = ± 1.
Note:
   The numbers (n) that end with a 4 or 6 have a trig. function of ± φ/2.
   The numbers (n) that end with a 2 or 8 have a trig. function of ± 1/(2φ).
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[8] *Reference - E.G.McClain –"The Pythagorean Plato: Prelude to the Song Itself"- p.23 ISBN 0-89254-010-9 –1984 - Publisher Nicolas-Hays, Inc.- York Beach, Maine-03910*

Listed here are the major angles and various angular combinations from the drawing of the Vitruvian man.

- ❑ 144° = 72° + 72°   Cosine 144° = -0.8090 = - φ/2
- ❑   72°   Cosine   72° = 0.3090 = 1/(2φ)
- ❑   54° = 27° + 27°   Sine   54° = 0.8090 =   φ/2
- ❑   27°   27° not a Phi φ function number
- ❑   81°   81° not a Phi φ function number

- ❑ 144° + 54° = 198°   Sine 198° = -0.3090 = -1/(2φ)
- ❑   27° + 81° = 108°   Cosine 108° = -0.3090 = -1/(2φ)
- ❑   81° + 81° = 162°   Sine 162° = 0.3090 = 1/(2φ)
- ❑   72° + 81° = 153°   153° not a Phi (φ) function number
- ❑   72° + 27° = 99°   99° not a Phi (φ) function number

Note: All odd numbers and sums that are odd numbers fail the Phi function rule. However, when an odd numbered angle is viewed as a *mirror image pair*, the double-angle is seen as an even number and may possibly obey the Phi function selection rule.

The two angles, 81°opposite 81°, form a *mirror image pair* that has a sum of 162°. This satisfies the Phi function rule just as does the angle 27° opposite 27° that form the 54°open leg stance of the Vitruvian man.