DEMONSTRATION OF THE BEAL CONJECTURE

BY THE THEOREM OF SEBÁ

By

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INTRODUCTION

The statement of the Beal conjecture is as follows:

If $A^x + B^y = C^z$, where A, B C, x, y and z are positive integers and x, y and z are all greater than 2, then A, B and C must have a cammon factor.

From the statement of the Beal conjecture, two points deserve to be discussed:

First, the members of the committee did not clarify whether A, B and C should be equal or different.

Second, if x, y and z should each be different. Clearly x, y and z cannot be equal, because otherwise it reduces to Fermat's equation, and this has already been demonstrated by Andrews Wiles, which has no positive integers positive solution. Based on this, it is our objective to present solution of positive integers to Beal's equation in various ways with, always a factor common to A, B and C, using Sebá's theorem, which will be stated and demonstrated below.

Theorem of Sebá. The equation $A^x + B^x = C^y$ has its solution in positive integers, with a common factor to A, B and C, always when

x and y are primes between then or (x, y) = 1

DEMONSTRATION

Bing $\mathbf{a^x + b^x = c^y}$, where **a**, **b** , **c**, **x** and **y** are positive integers

Multiplying both members of $\mathbf{a^x + b^x = c^y}$ **by** $\mathbf{(a^x + b^x)^y}$, results in:

$$\mathbf{(a^x + b^x)\,(a^x + b^x)^y = (a^x + b^x)\,c^y} \qquad \mathbf{(1)}$$

With $\mathbf{c^y = a^x + b^x}$, and direct substitution of the value of $\mathbf{c^y}$ in (1), results in:

$$\mathbf{(a^x + b^x)\,(a^x + b^x)^y = (a^x + b^x)^{y+1}}$$

or

$$\mathbf{a^x\,(a^x + b^x)^y + b^x\,(a^x + b^x)^y = (a^x + b^x)^{y+1}} \qquad \mathbf{(2)}$$

Where, $\mathbf{(a^x + b^x)}$ is the common factor to A, B and C.

When we attribute values (2) to **a** and **b**, with a = b, a<b or a>b, we found A, B and C are always positives and integers, and $\mathbf{(a^x + b^x)}$ a common factor to A, B and C.

METHOD OF RESOLUTION OF SEBA'S EQUATION

For example, dividing a square in two cubes varies in different ways,

$$\mathbf{A^3 + B^3 = C^2}\ .$$

When x=3, substituting in (2), will result in:

$$\mathbf{a^3\,(a^3 + b^3)^y + b^3\,(a^3 + b^3)^y = (a^3 + b^3)^{y+1}} \qquad \mathbf{(3)}$$

In the equation $\mathbf{A^3 + A^3 = C^2}$, the left member is in the power of three and the right member in the power of two, therefore we must find values for **y** and **y+1**, and when possible decompose **y** in powers of

three and **y+1** in powers of two. That is only possible when **y** and **y+1** are multiples of 3 and 2 respectively.
So **y = 12n - 9** and **y+1 = 12n - 8.** Substituting the values of **y** and **y+1** in (3), resulting in:

$$\mathbf{a^3(a^3 + b^3)^{12n - 9} + b^3(a^3 + b^3)^{12n - 9} = (a^3 + b^3)^{12n - 8}} \qquad \mathbf{(4)}$$

For **n = 1** and **a = b = 1**. Substituting the values of n, a and b in (4), results in:

$$\mathbf{2^3 + 2^3 = (2^2)^2}$$

Solution: A = B = 2 and C = 4. The common factor for A, B and C is

$$\mathbf{(a^x + b^x) = (1^3 + 1^3) = 2.}$$

Another solution is **n** = 2, **a** = **b** = 1. Substituting in (4), results in:

$$\mathbf{(2^5)^3 + (2^5)^3 = (2^8)^2}$$

Solution: A = B = 32 and C = 256. The common factor for A, B and C is 2.

Another solution is to choose **n** = 1, **a** = 1 and **b** = 2. Substituting in (4), results in:

$$\mathbf{9^3 + 18^3 = (81)^2}$$

Solution: A = 9, B = 18 e C = 81. The common factor for A, B and C is $\mathbf{(a^x + b^x)} = \mathbf{(1^3 + 2^3) = 9.}$ And so on.

For example, dividing a bi-quadratic in two cubes in various different ways, so

$$\mathbf{A^3 + B^3 = C^4} \; .$$

Known that x = 3, substituting in (2), results in the same equation of (3).

For the given equation, $\mathbf{A^3 + B^3 = C^4}$**,** the left member is in the power of 3 and, the right member in the power of 4, therefore, we must round values for **y** and **y+1**, and when possible decompose **y** in powers of three and **y+1** in powers of four. That is only possible when **y** and **y+1** are **multiples** of **3** and **4** respectively. In that case, **y = 12n - 9** and **y+1 = 12n - 8.** Substituting the values of **y** and **y+1** in **(3)**, results in the same equation as **(4)**. Choosing **n = 1** and **a = b = 1** and substituting in (4), the results is:

$$\mathbf{2^3 + 2^3 = 2^4}$$

Solution: A = B = C = 2. The common factor is 2. Choosing, for example,
a = b = 2 and **n = 1**, and substituting in **(4)**, results in:

$$\mathbf{2^3\,(16)^3 + 2^3\,(16)^3 = 16^4}$$

$$\mathbf{(2x16)^3 + (2x16)^3 = 16^4}$$

Solution: A = B = 32 and C = 16. The common factor to A, B and C is $\mathbf{(a^x + b^x)}$ = $(2^3 + 2^3) = 16$. And so on and so forth.

Let us, for example, to divide a cube into two cubes that is: $A^3 + B^3 = C^3$

As the exponents of the given equation are not prime among them, it is impossible to divide a cube into two cubes. Also it is not possible to divide one bi-square into two bi-square, a square into two bi-square, or a 5th degree power into two of 5th degree , and so on.

POSITIVE INTEGER SOLUTION TO BEAL'S EQUATION WHERE A, B AND C HAVE A COMMON FACTOR

Based on the results shown above it is possible to conclude that:
as the equation $\mathbf{A^x + B^x = C^y}$ **(Sebá's equation)** when A, B, C, x and y are positive integers and, y primes between them, has a factor $\mathbf{(a^x + b^x)}$ common to A, B and C, so the equation $\mathbf{A^x + B^y = C^z}$ **(Beal's equation)** also has the factor
$\mathbf{(a^x + b^x)}$ common to A, B and C. This will be shown as follows.

To find the solutions in positive integers for Beal's equation, based on Sebá's equation, just write Beal's equation in the following way:

$$\mathbf{A^{xk} + B^{ym} = C^z} \qquad \mathbf{(5)}$$

where xk = ym. For example: to find the solution in positive integers do the equation: $\mathbf{A^6 + B^3 = C^7}$. Let **k** = 1 and **m** = 2. Therefore the equation gives $\mathbf{A^6 + B^6 = C^7}$ (Sebá's equation)

The numbers y = 42n -36 and y+1 = 42n -35 are multiple of 6 and 7, respectively. Substituting for **a** and **b**, in equation (2), for 1:

$$\mathbf{2^{42n - 36} + 2^{42n - 36} = 2^{42n - 35}}$$

If n = 1, $2^6 + 2^6 = 2^7$ or $2^6 + (2^2)^3 = 2^7$
Solution: A = C = 2 and B = 4. The common factor among A, B and C is
$a^x + b^x = 1^6 + 1^6 = 2$.

If $A^3 + B^2 = C^7$, then $(2^2)^3 + (2^3)^2 = 2^7$
Solution: A = 4, B = 8 and C = 2.

If $A^6 + B^2 = C^7$, then, $2^6 + (2^3)^2 = 2^7$.
Solution: A = C = 2 and B = 8.

In order to obtain another solution, let us take **n** = 1, **a** = 1 and **b** = 2 in equation (2), and then:

$$\mathbf{65^6 + (2x65)^6 = 65^7}$$

$$\mathbf{65^6 + (130^2)^3 = 65^7}$$

Solution: A = C = 65 and B = 16900. The common factor among A, B and C is: $a^x + b^x = 1^6 + 2^6 = 65$. If $A^3 + B^2 = C^7$, then, $4225^3 + (130^3)^2 = 65^7$.
If $A^3 + B^2 = C^7$, then, $(4225)^3 + (130^3)^2 = 65^7$.
Solution: A = 4225, B = 130^3 and C = 65 or A = 65^2, B = 2x65 and C = 65.
If $A^6 + B^2 = C^7$, then, $65^6 + (130^3)^2 = 65^7$
Solution: A = C = 65 and B = $(2x65)^3$

As an example, find integer solutions for the equation $A^4 + B^8 = C^9$. Take **k** = 2 and **m** = 1. Then from equation (5), the equation $A^8 + B^8 = C^9$ (Sebá's equation) is obtained. The numbers y = 72n - 64 and y+1 = 72 - 63 are multiple of 8 and 9, respectively. Substituting for **a** and **b** in equation (2), for **1** the following equation is obtained:

$$\mathbf{2^{72n - 64} + 2^{72n - 64} = 2^{72n - 63}}$$

If **n** = 1, then, $2^8 + 2^8 = 2^9$ or $(2^2)^4 + 2^8 = 2^9$.
Solution: A = 4 and B = C = 2. The common factor among A, B and C is 2.

If $A^2 + B^8 = C^9$, then, $(2^4)^2 + 2^8 = 2^9$.
Solution: A = 16, B = C = 2. The common factor among A, B and C is 2.

If $A^2 + B^8 = C^3$, then, $(2^4)^2 + 2^8 = (2^3)^3$.
Solution: A = 16, B = 2 e C = 8. The common factor among A, B and C is 2.

If $A^2 + B^4 = C^9$, then, $(2^4)^2 + (2^2)^4 = 2^9$.

Solution: A = 16, B = 4 and C = 2. The common factor among A, B and C is 2.

Others integers and positive solutions for the equation $A^3 + B^4 = C^5$, can be obtained by choosing **n** = 1, **a** = 1 and **b** = 2 in (2), and then:

$$\mathbf{(1 + 2^{12})^{100n - 76} + 2^{12}(1 + 2^{12})^{100n - 76} = (1 + 2^{12})^{100n - 75}}$$

$$\mathbf{(1 + 2^{12})^{24} + 2^{12}(1 + 2^{12})^{24} = (1 + 2^{12})^{25}}$$

$$\mathbf{((1 + 2^{12})^{8})^{3} + (2^{3})^{4}((1 + 2^{12})^{6})^{4} = ((1 + 2^{12})^{5})^{5}}$$

$$\mathbf{((1 + 2^{12})^{8})^{3} + (2^{3}(1 + 2^{12})^{6})^{4} = ((1 + 2^{12})^{5})^{5}}$$

Solution: $A = (1 + 2^{12})^8$, $B = 2^3(1 + 2^{12})^6$, and $C = (1 + 2^{12})^5$.

The common factor among A, B and C is:

$$(a^x + b^x) = (1^{12} + 2^{12}) = 1 + 2^{12}$$

As another example, find integer solutions for the equation $A^3 + B^4 = C^5$. Take k = 4 and m = 3 and from equation (5) the typical Sebá's equation $A^{12} + B^{12} = C^5$ is obtained.

Replacing **a** and **b** by 1 in equation (2), the following equation is obtained

$$\mathbf{2^{60n - 36} + 2^{60n - 36} = 2^{60n - 35}}$$

If n = 1, then, $2^{24} + 2^{24} = 2^{25}$ or $(2^8)^3 + (2^6)^4 = (2^5)^5$

Solution: A = 256, B = 64 and C = 32. The common factor among A, B and C is 2.

CONCLUSION.

From the results obtained from Sebá's theorem and equation, the following was concluded:

Even though the committee members did not clarify whether A, B and C should be equal or different, and if x, y and z should be different, we demonstrated that: every time a specific equation of Beal reduces to Sebá's equation, the Beal's equation has a positive integer solution. Also, the factor $(a^x + b^x)$ that is common to A, B and C is Beal's equation.

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